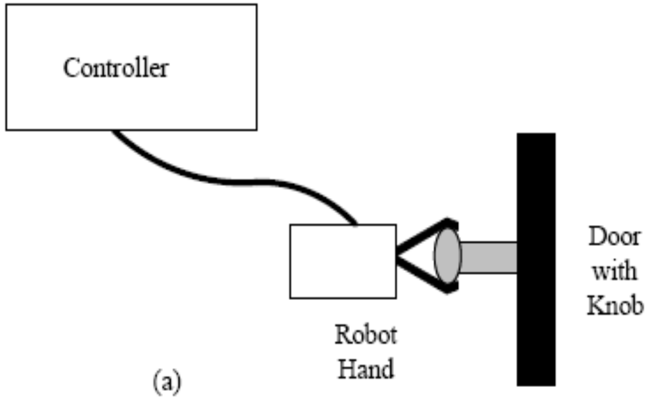
GRASP Laboratory

General Robotics & Autonomous Systems and Processes

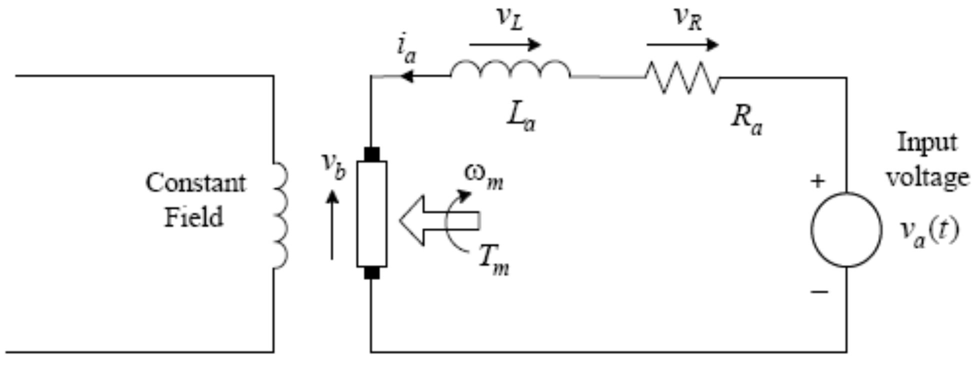
Mechatronic Modeling and Design with Applications in Robotics

Linear Graph

Robotic Door Opener

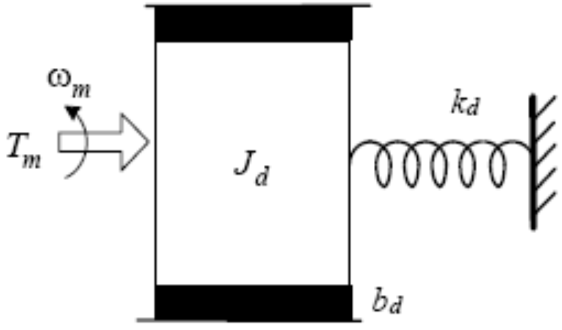


(a)

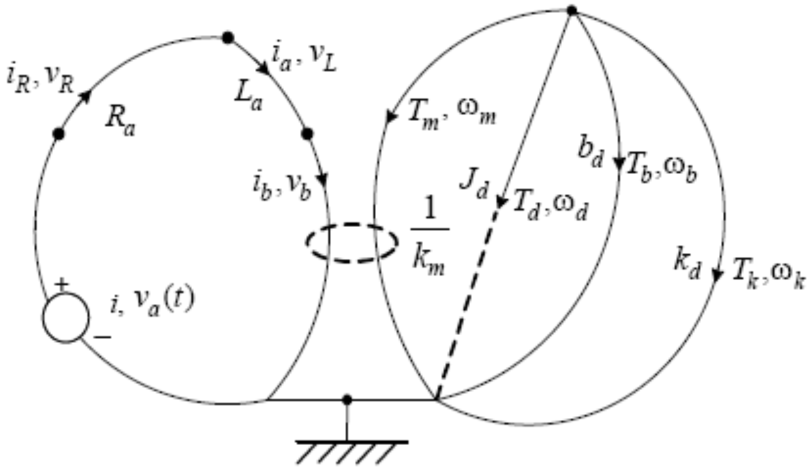


Armature Circuit

(b)



(c)



Linear Graph

- Provide a unified (multi-domain) modeling tool
- Provide a graphical representation of a model (allow visualization of system structure before model formulation)
- Use interconnected line segments (branches) that represent elements
- Help identify similarities (in domain, structure, behavior, etc.) in systems
- Facilitate development of computer-based modeling tools and software (systematic, graphical)
- Applicable for lumped-parameter systems

- *Note*: "Linear" → "line" (Can model nonlinear systems with nonlinear constitutive equations)

Analogies and Constitutive Relations

System Type	Constitutive Relation for		
	Energy Storage Elements		Energy Dissipating Elements
	A-Type (Across) Element	T-Type (Through) Element	D-Type (Dissipative) Element
Translatory-Mechanical v = velocity f = force	Mass (Newton's 2nd Law) m = mass	Spring (Hooke's Law) k = stiffness	Viscous Damper b = damping constant
Electrical v = voltage i = current	Capacitor C = capacitance	Inductor L = inductance	Resistor R = resistance
Thermal T = temperature difference Q = heat transfer rate	Thermal Capacitor C_t = thermal capacitance	None	Thermal Resistor R_t = thermal resistance
Fluid P = pressure difference Q = volume flow rate	Fluid Capacitor C_f = fluid capacitance	Fluid Inertor I_f = inertance	Fluid Resistor R_f = fluid resistance

Through-Variables and Across Variables

- A branch has an ordered pair [*through-variable*, *across-variable*] (f , v)
  - The product is "Power"

Note: The relationship between f and v can be nonlinear in general.

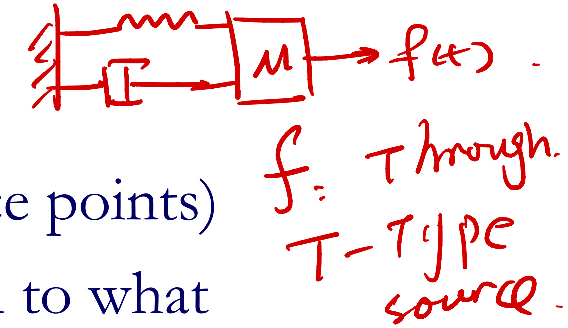
System Type	Through	Across
Electrical	Current	Voltage
Mechanical	Force/Torque	Velocity/Angular Velocity
Thermal	Heat Transfer	Temperature
Hydraulic/pneumatic	Flow Rate	Pressure

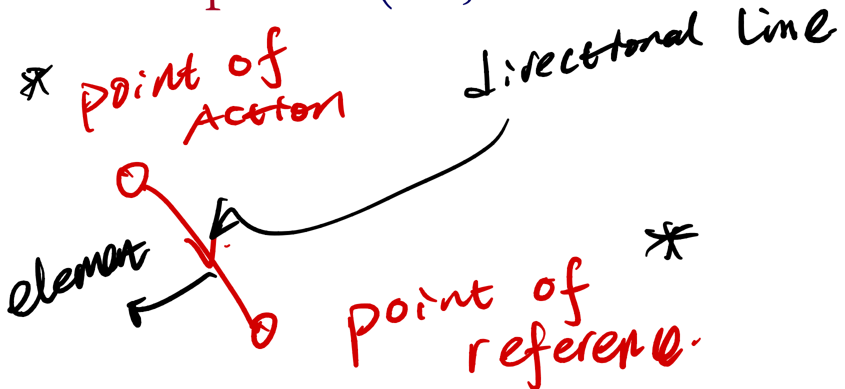
Note: Force is transmitted through the element with no change. Velocity is measured across the element, relative to one end.

- Through variable is the same at input and output of element
- Across-variable is given with respect to a reference point

Sketching a Linear Graph

1. Identify the energy storage elements, energy dissipation elements, and source elements in the system (1-port elements, each represented by 1 branch)
2. Identify any multi-port elements (e.g., transformers, gyrators)
3. Identify the terminals of each element (i.e., action points and reference points)
4. Recognize how the elements are interconnected (series or parallel and to what elements?) and sketch a schematic diagram (e.g., circuit diagram)
5. Starting from a convenient node point (typically, ground reference) draw a branch (typically, for a source), link it to another branch through a node, and so on, to form a loop
6. Repeat Step 5 until the entire system is completed (i.e., all the elements in the system are included and connected)



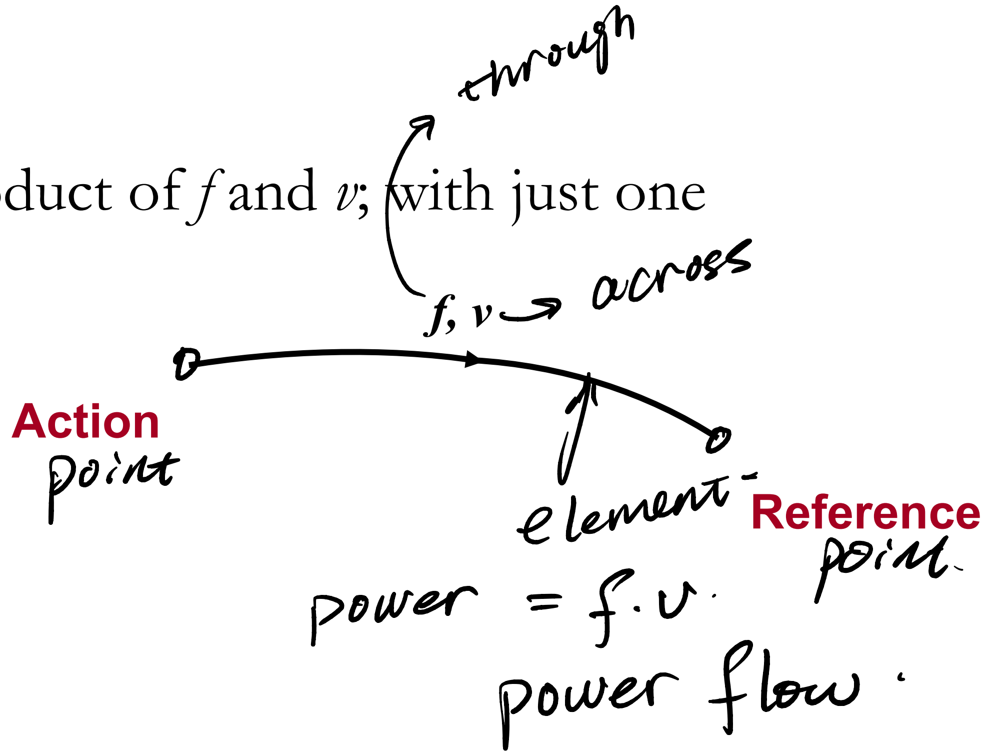


Single-Port Elements

Port: Place where the element exchanges energy/power with environment (other connected elements) with respect to a reference (because across-variable is *wrt* reference).

A Single-Port (*Single Power or Energy Port)* Element:

- Represented by a single branch (line segment)
- Has a through-variable *f* and a corresponding across-variable *v*, given as the ordered pair *f*, *v* on the branch
- The relationship between *f* and *v* can be nonlinear
- Possesses only one power variable, which is the product of *f* and *v*; with just one "power port" hence the name
- Has two terminals (action and reference)

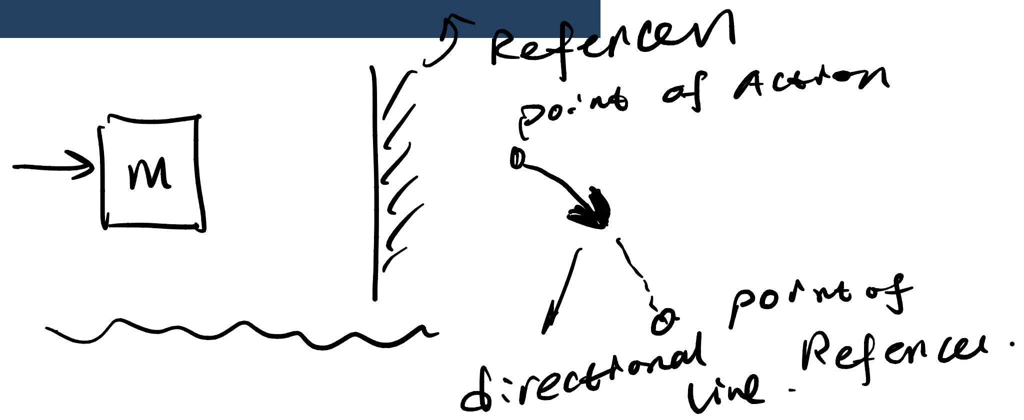


Sign Convention (Single-Port Element)

One end of branch: Point of action

Other end of branch: Point of reference

(Somewhat arbitrary, can reflect the physics of system)



Oriented branch ➔ Direction is assigned using an arrowhead ➔ Positive direction of power flow ➔ "Into" element at point of action, and "out of" element at point of reference (exception: "source") ➔ Arrowhead of branch is pointed toward point of reference (exception: T-type source)

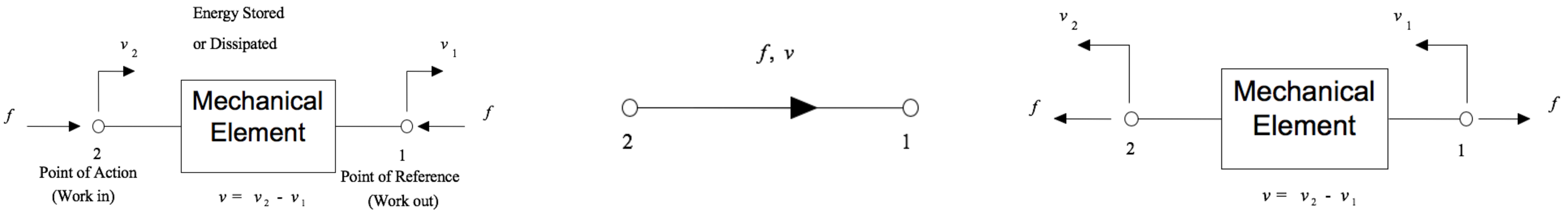
Note: Arrow does not represent the positive direction of f or v

Across-variable: Given relative to point of reference

Pre-establish positive direction of any one of f and v: → Positive direction of the other variable

Convention: Assign direction of v and power flow the same at point of action (Figure (a) is preferred over Figure (c), in the case of mechanical element).

Sign Convention (Single-Port Element)



Note: Reversing the directions of both *f* and *v* won’t affect power flow direction (See (a) and (c); Convention (a) is preferred

(A) Work done “on” element at point of action (by an external device) is positive

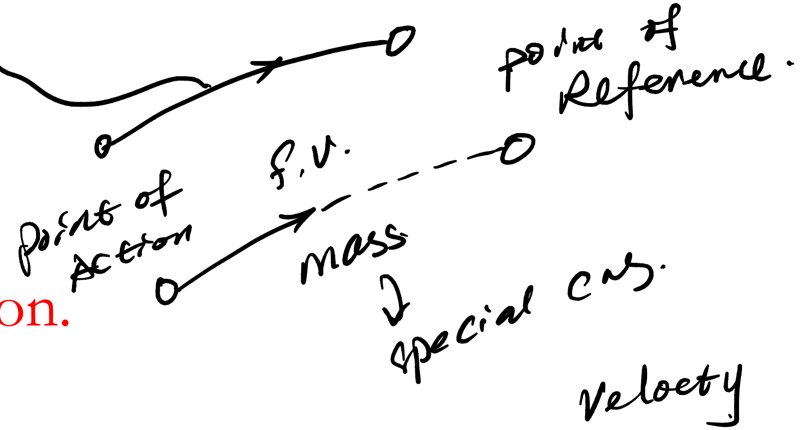
(B) Work done “by” element at point of reference (on an external load) is positive.

Note: (A)-(B) ➔ Stored in element (e.g., kinetic, potential) ➔ Has capacity to do work or dissipated (damping) through mechanisms manifested as heat transfer, noise, deformation, etc.

Note: Figure shows a mechanical element. But the same argument applies to an electrical element except that current “*f*” is out of the element in at reference point in (a), unlike the “reaction force” which is into the element (but for the next “connected” element it is “into” as with electrical current).

Linear Graphs of Mechanical Elements

Three Single-port Elements: Mass, Spring, Dashpot/Damper

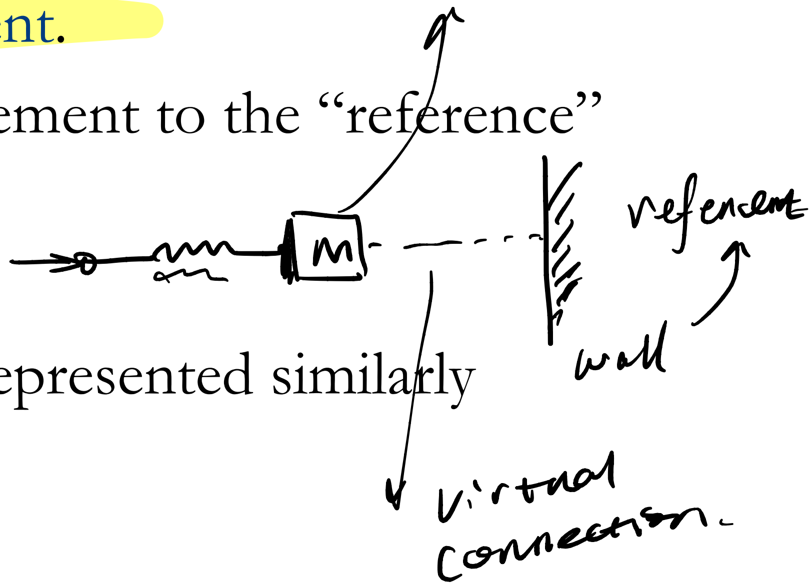


Corresponding rotary elements are easy to visualize –

f = applied torque; v = relative angular velocity in same direction.

Note: Linear graph of an inertia element has a broken line segment.

Force does not physically travel from "action" terminal of the element to the "reference" terminal (inertial reference) but "felt" there.

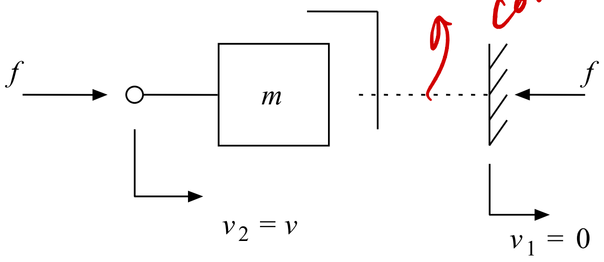


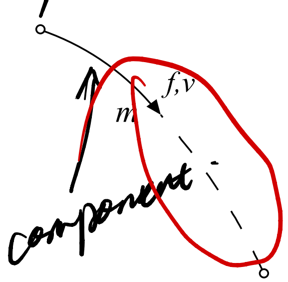
Note: Analogous electrical, fluid, and thermal elements may be represented similarly

Linear Graphs of Mechanical Elements

Mass

Energy Storage Element (Inertia/Mass) Mass $= m$



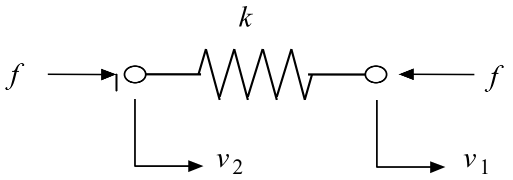


point of Reference.

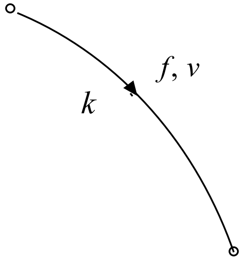
No physical connection between mass & Reference

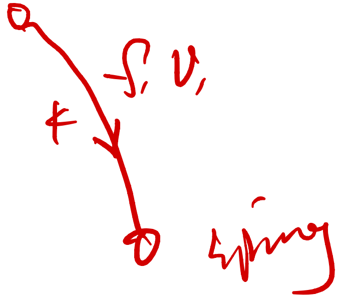
Spring

Energy Storage Element (Spring) Stiffness $= k$



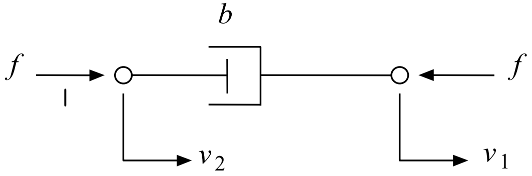
$v = v_2 - v_1$



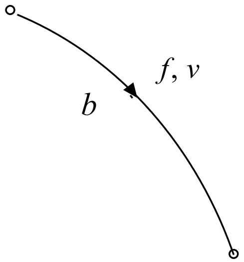


Damper

Energy Dissipation Element (Damper)
Damping Constant $= b$

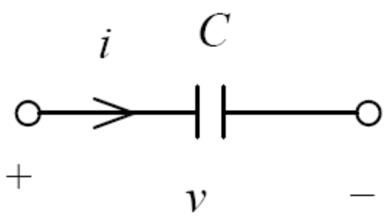


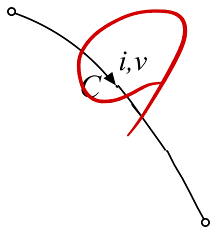
$v = v_2 - v_1$



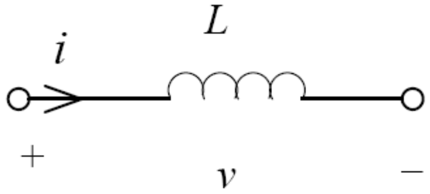
Linear Graphs of Electrical Elements

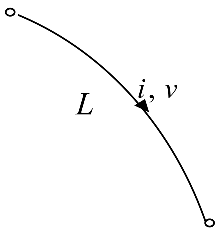
Energy Storage Element (Capacitor)
Capacitance = C



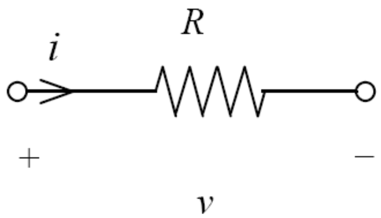


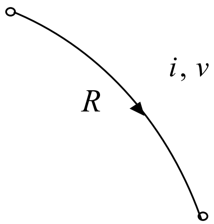
Energy Storage Element (Inductor)
Inductance = L

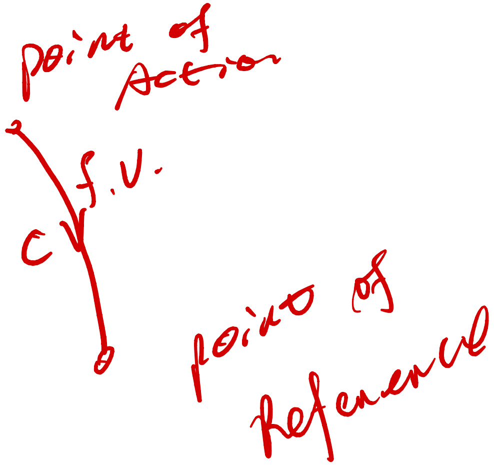




Energy Dissipation Element (Resistor)
Resistance = R

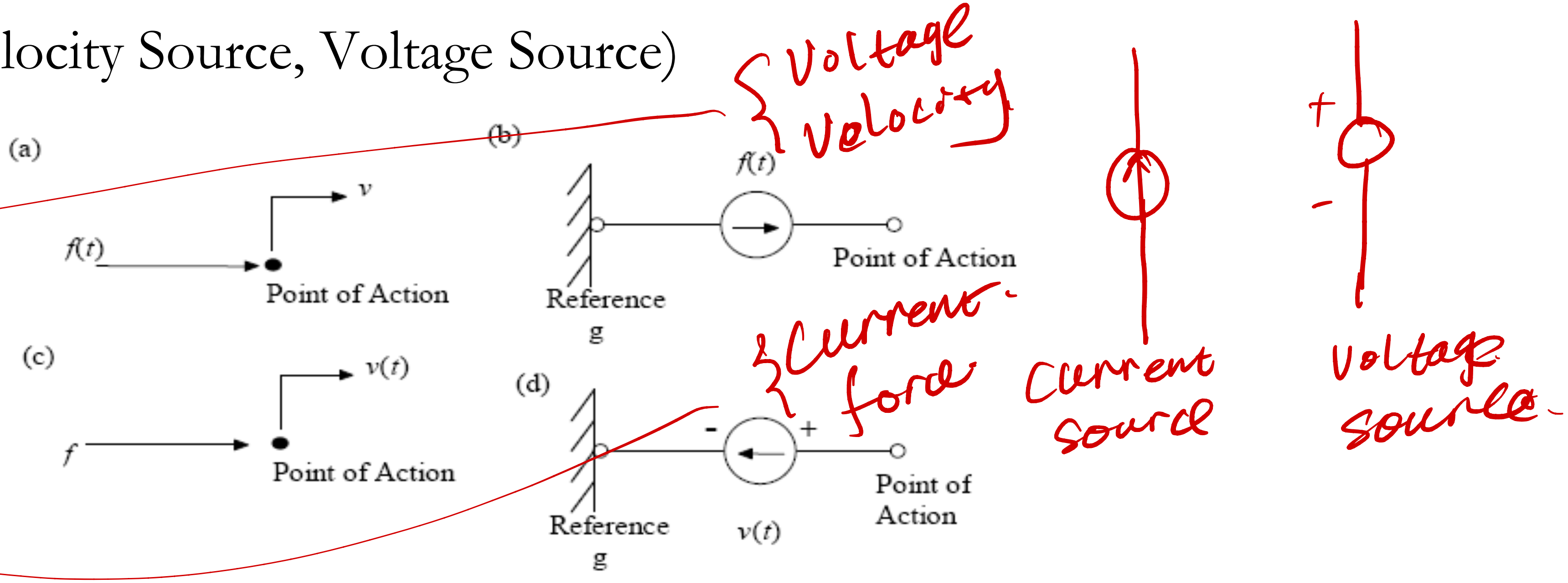






Source Elements

- T-Type Source (e.g., Force Source, Current Source)
- A-Type Source (e.g., Velocity Source, Voltage Source)



(a) T-Type Source (through-variable input); (b) Linear graph representation

(Arrowhead: Positive direction of through variable)

(c) A-Type Source; (d) Linear graph representation

(+ sign: point of action; – sign: point of reference; Arrow head: direction of across variable drop)

Effects of Source Elements

For an Ideal Source: Source variable (independent variable) is unaffected by dynamics of the connected system. But co-variable (dependent variable) will change.

Source elements can serve to inhibit interactions between systems.

Dynamic behavior of system is not affected by connecting a new system:

(a) in series with an existing T-source

(b) in parallel with an existing A-source.

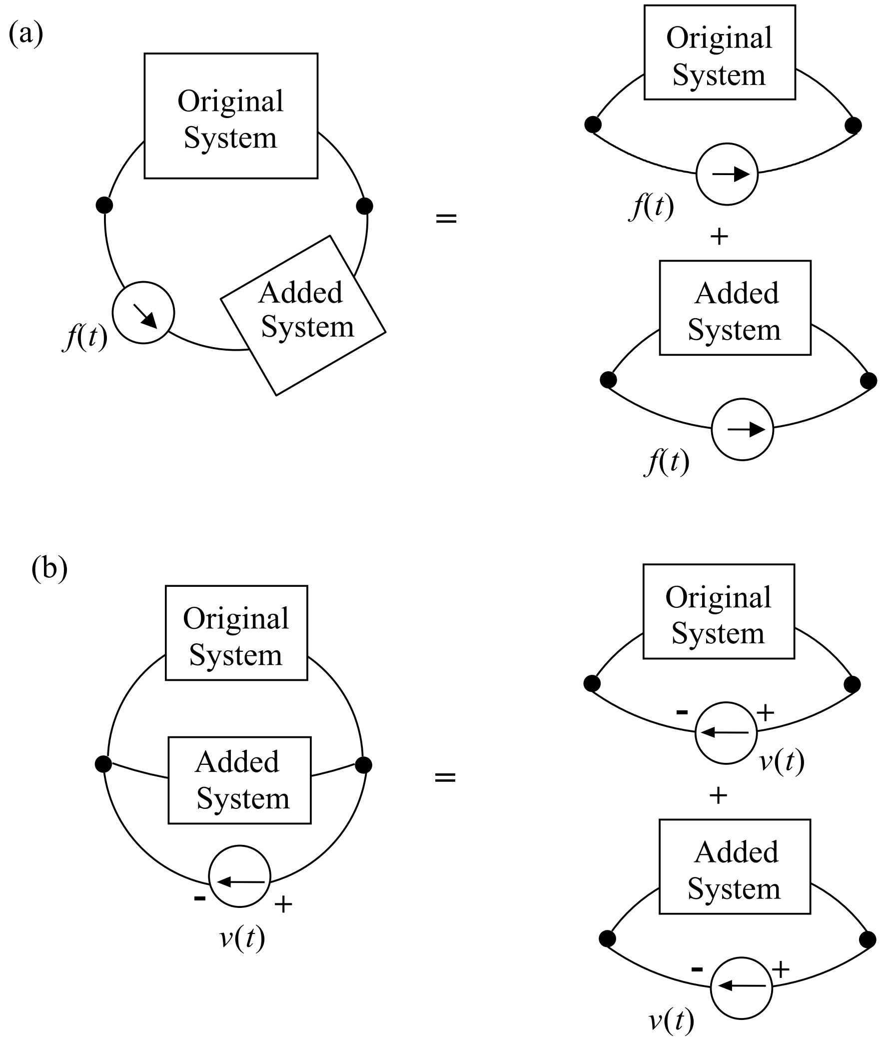
The modified system may be treated (analyzed) as two separate uncoupled systems driven by the same input source

Order of the overall system = sum of the orders of individual subsystems.

Effects of Source Element

(a) Two systems connected in series to a T-source

(b) Two systems connected in parallel to an A-source.



Transformer (Two-Port Element)

Ideal Transformer: Converts the across-variable at the input port into the across-variable at the output port (and the corresponding through-variable at the input port into the through-variable at the output port) without any energy storage or dissipation.

Mechanical Transformer:

Arrows on each branch → Positive direction of power flow (from action to reference; force × velocity > 0 into the branch at action point)

v_i and f_i = velocity and force at input port

v_o and f_o = velocity and force at output port

For a linear transformer, transformation parameter r is given by $v_0 = rv_i$

Conservation of power: $f_i v_i + f_o v_o = 0$ → $f_0 = -\frac{1}{r} f_i$

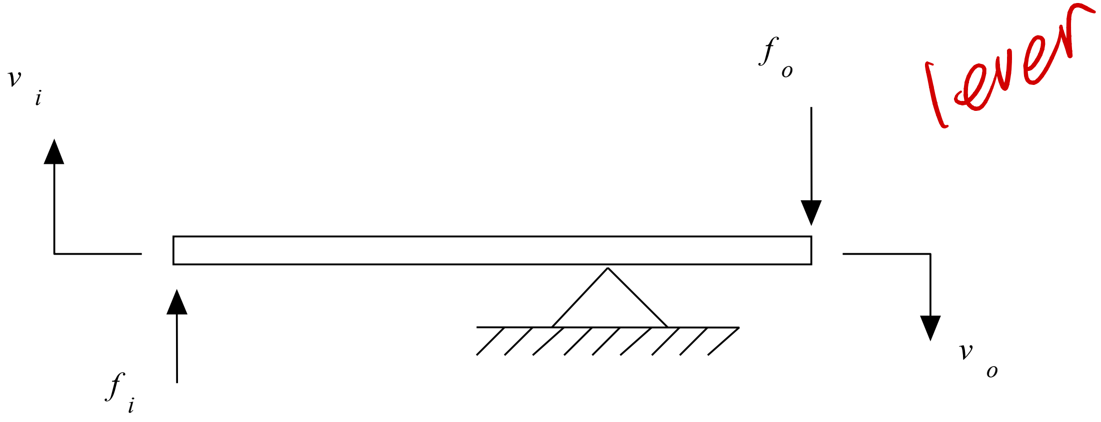
Note: r is a non-dimensional parameter if the domains of the two sides of the transformer are the same.

(i) and (ii): Constitutive relations for transformer

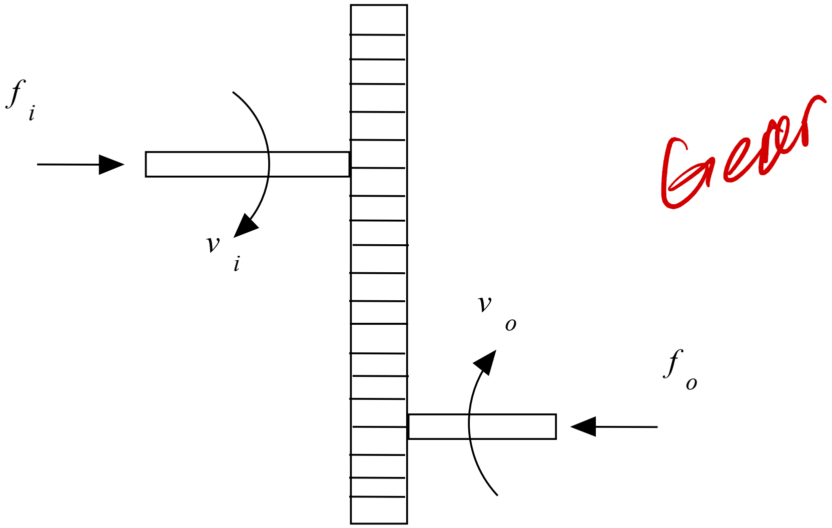
Note: Transformers in other domains (e.g., electrical transformers, transducers, fluid transformers) and mixed domains (e.g., dc motor) may be treated similarly (neglect dissipation and storage)

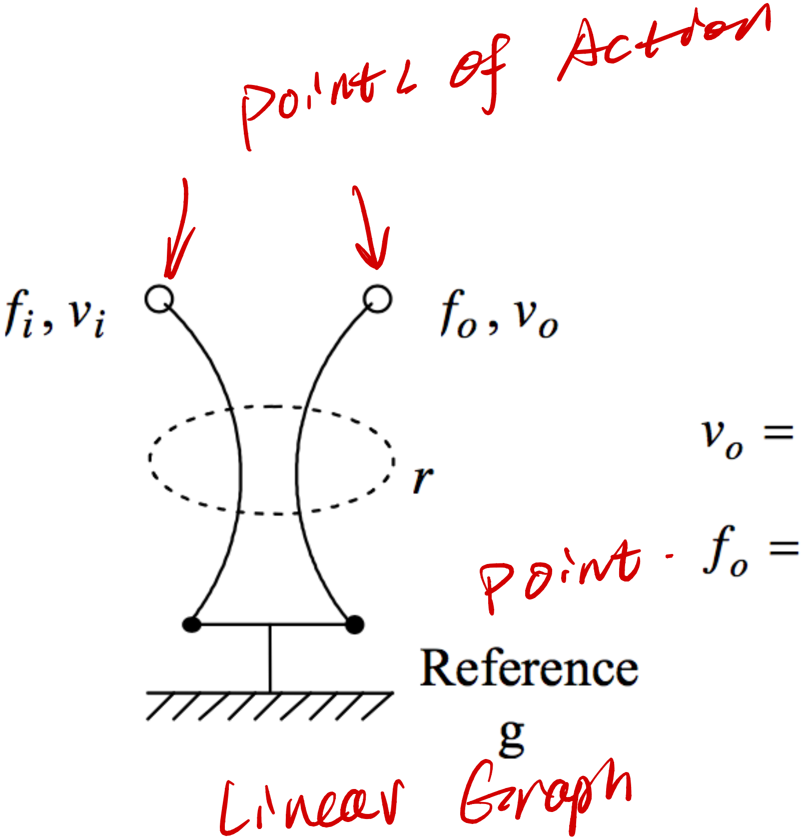
Mechanical Transformer

(a)



(b)

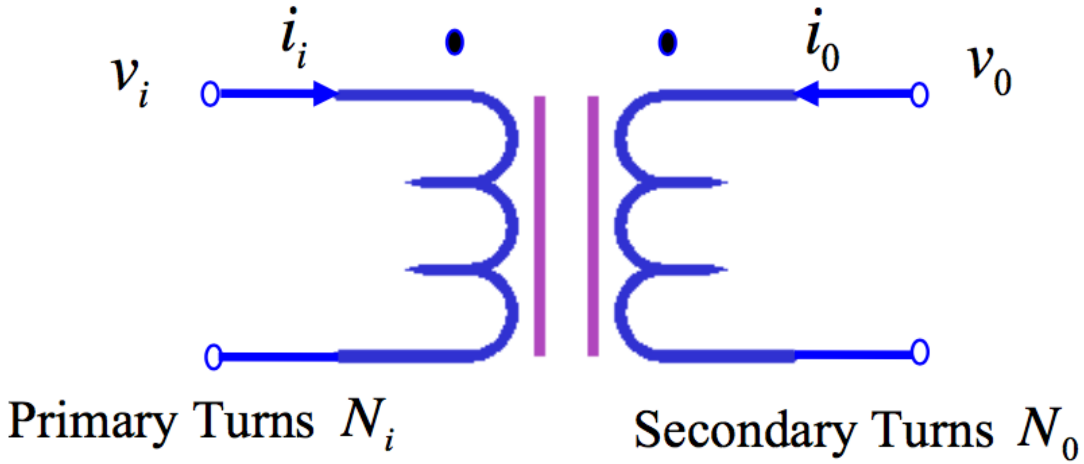




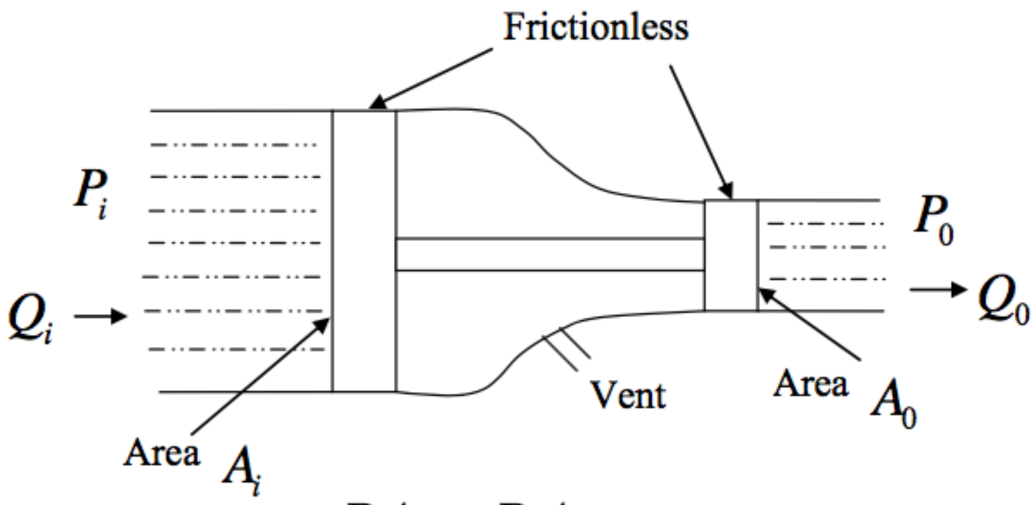
$$v_o = r v_i$$

point. $$f_o = -\frac{1}{r} f_i$$

Electrical and Fluid Transformers



$$r = \frac{N_0}{N_i} = \frac{v_0}{v_i}$$



$$P_i A_i = P_0 A_0$$

$$r = \frac{A_i}{A_0} = \frac{P_0}{P_i}$$

The End!!